Portfolio Selection and Optimal Investment in Nigerian Stock Market

Blessing E. Chukwuemeka and G.C.E. Mbah
Department of Mathematics
Chukwuemeka Odumegwu Ojukwu University, Uli
and University of Nigeria, Nsukka
blessingchukwuemeka1@gmail.com and Godwin.mbah@unn.edu.ng

D.O.I: 10.56201/ijasmt.v8.no3.2022.pg92.106

Abstract
The work portfolio selection and optimal investment in Nigeria stock market, took consideration of the financial statements of seven companies that were floated in the Nigeria Stock Exchange from 2012-2016 in our linear programming problem, and took cognizance in their returns (dividend) and risk factors. In this work we looked at the returns and risks of each portfolio selected, we solved them using simplex method of solving optimization problem and obtained the result for the primal problem which contained, how much an investor can invest and the return he will get by investing such amount. Consequently, the returns from the best three of the selected companies were adopted to guide an investor in investment decision.

Keywords: *Portfolio Selection, Investment, Stock and Market, simplex method techniques*

Introduction

Portfolio is bringing together all investments owned by the same person or organization. These investments can include stocks (which are investments in individual business), bonds (which are investments in debt that are designed to earn interest), and mutual funds (which are essentially pools of money from many investors that are invested by professionals). Portfolio is bringing together all investments owned by the same person or organization. These investments can include stocks (which are investments in individual business), bonds (which are investments in debt that are designed to earn interest), and mutual funds (which are essentially pools of money from many investors that are invested by professionals).

Portfolio can also be defined as a combination of financial risk such as stocks, bonds and cash.

Portfolio can be held by individual investors or managed by financial professionals, hedge funds, banks and other financial tolerance.

The monetary value of each asset may influence the risk/reward ratio of the portfolio and is referred to as the asset allocation.

According to freelancing website, Portfolio can also be the past work which you had done or

shares which you have in any company. An investment is defined as an asset or item that is bought with the hope of making profit or will appreciate in the future.

In economics, investment is creation of capital or good(s) capable of producing other goods or services. Investment requires one's money, time, and effort and so as to make profit or get an advantage.

Investment can be in in form of stocks, bonds, mutual funds, interest-bearing accounts, land, derivatives, real estate, art work, old comic books, Jewellery- anything that an investor believes can fetch him money usually in the form of interest or rents.

Generally, to invest means to allocate money with the expectation of some benefits in the future. The benefit from investment is called a return.

The return may consist of capital gains or investment income or a combination of the two.

Investments are grouped into three types. They include:

Ownership investments: these include the investments in stocks, business, real estate etc.

Lending investments: these include bonds and your savings account.

Cash equivalents, which include money market bonds (as good as cash). This means that they can easily be converted back to cash.

Education can often be called an investment because it helps one to earn high income many times.

RETURN ON INVESTMENT

Return on investment is a performance measure that is used to determine the efficiency of an investment or to compare the efficiency of a number of different investments. This measure the amount of return on an investment relative to the investment's cost.

The investment return r can be calculated when the benefit of an investment is divided by the cost of the investment.

This can be represented as:

$$\mathbf{r} = \frac{P_T - P_0}{P_0} \times 100$$

P_T = Total amount from the investment at final time (which includes the dividends and the interest)

P_0= is the cost of the investment at initial time

r = return on investment

This definition of the return on investment and the means of calculating a return on investment can as well be modified to suit the situation.

For example, in assessing a return on a real estate investment, one might use the initial purchase price of the property as the "cost of investment" and the ultimate sale price as the gain or total amount at the final time from investment, though this did not account for all the intermediary costs, like renovations property, taxes and real estate agent fees.

In financial market, return can also be referred to as gain or loss in the investment of an asset (Chandra et al 2013) and is normally expressed in percentage. In this case r signifies gain when it is positive and loss when it is negative. Zero means nether gain or loss in the investment.

Determination of The Return On Investment

The return on investment can be determined using the rate of return. The rate of return is a profit on an investment over a period, which is expressed as proportion of the original investment. The time period is mainly a year; therefore, the rate of return is referred to as an annual return.

Return can be determined based on the type and the period

$$\text{Return on Asset} = \frac{Annual\ Net\ income}{Average\ Total\ Asset} \quad \text{or} \quad \frac{Revenue}{total\ asset}$$

Return can also be measured from the companying's annual financial statement to know their performance and risk involved in investing in the company.

Organization's profit can be measured using its rate of return on total asset.

$$r = \frac{profit\ after\ tax}{total\ asset}$$

RISK

Risk is defined as the degree of uncertainty of return on an asset which signifies the possibility of loss in the investment.

In finance, risk is the possibility of actual return being lower than expected return. It involves the possibility of losing some or all the original investment. The risk can either be zero, which means that the asset is risk free, but if the risk is positive, it tells us that the asset is risky.

So, if the asset is risk- free, then the future value of the asset is certain while uncertain when risky (Chandra et al 2013).

An example, fixed deposit is a risk-free asset and share of stock or share of a mutual fund is classified as risky asset. Risk can be caused by many things, like the price for raw materials, the lapsing of deadlines for construction of a new operating facility, disruptions in a production process, emergence of a serious competitor on the market, the loss of key personnel, the change

of a political regime, or national disasters.

Conceptual Clarification

Over the years there have been various studies by scholars on portfolio selection. These Portfolio selection methods and processes are the pillar on which the Modern Portfolio Theory (MPT) propounded **by Harry Markowitz in 1952** is built on. The model is a system of collecting stocks on investment of different companies, and optimization of capital allocation to several securities. Because a portfolio is a collection of securities, the investor must have to take the decision on the portfolio to invest. Thus, the decision of the investor is equivalent to selecting an optimal portfolio from available portfolios; which is often referred to as the portfolio selection.

Dhaene, *et al* 2005 described the portfolio selection as a process used to identify the best allocation of securities for an investor with a given savings or consumption behaviour over a given horizon. A good portfolio is more than a long list of good stocks and bonds.

Theoretical Framework

This theory of investment was enough to regard as expected values for determining future stock value that is, when an investor wants to maximize the profit, it was enough considering only one stock. Then Markowitz said that risk should not be neglected when considering the portfolio. In (1991) Markowitz came up with another new theory which associated risk with variance. He analyzed stock under the historical data in a risky environment and studied it to minimize risks and maximizing return.

Fanelli and Lee (2011) sees mean variance as a technique that uses expected returns, variance and co-variance of individual investments to analyze risk returns trade-off of combinations of assets within an investment portfolio. This technique of theirs works mainly in the short term and long-term investors' decisions (where the portfolio may be re adjusted several times during the planning horizon).

Mark (2002-2018) proposed that one way to measure risk is to calculate the variance and standard deviation of the distribution of the returns.

Onwukwe (2016) measured risk factor of companies as total indebtedness of the company divided by total asset i.e. $\frac{Total\ Indebtedness}{Total\ Asset}$

In this work, liability of companies is seen as a measure of risk factor of the cooperation, which was calculated as **total liability of companies divided by the total asset of the company or total current liability divided by total asset**. i.e. $\frac{Total\ liability\ of\ companies}{Total\ Asset}$

Return measurement:

Return is the profit got from investment. Return and expected return have been measured in

different ways by different researchers.

Lee (1977) Lichtenberger and Kraus (1976), made an alternative study about distribution of return in portfolio theory. The future returns of investors using this model to Steinbach (2001) are overcome using probabilities in determining the expected returns of combinable portfolio securities.

Von Neumann- Morgenstern (1947) proposed the choice of a set of portfolios as efficient to maximize the expected utility of investors.

Bai et al (2009) proposed new bootstrap-corrected estimations for optimal return and its asset allocation and proved that boot strap corrected estimates are proportionally consistent with their theoretic counterparts.

Then in this work dividend is considered as a return from investment on share.

Methodology

This study is technical financial analysis which used the financial statements of seven companies floated in the Nigerian Stock Exchange over a 5-year period (2012-2016). The choice of 2016 is to determine the annual profit and liability. The consideration of 2016 is predicated on exploring the impacts of recession on the actual or real returns of the companies and the liabilities during the period of recession. The analysis in this work is geared towards choosing the proper stock to invest on while forecasting the future performances of these companies. Financial Statement analysis can be referred as establishment of relationship between the financial statements of two or more companies to arrive at a conclusion of what to invest on, and thus, emphasizes on the importance of the financial statement in decision making process.

Financial Statement is an investment instrument of analysis which highlights the performances of companies over time within a period in recent past and assists an investor in determining how the company can perform in a near future and as such, to decide how and what to invest on. Financial Statements are historical, because it exposes the operation and outcome of a company businesses tabulated in a way to show the cash flow, profitability, assets and liabilities. On the other hand, financial forecast basically considers the future performances to get an optimal return on investments and in turn minimizes greatlys risks taking cognizance of how the output of interested stock has performed in recent past. It is noteworthy that financial statements are a valuable tool that provides guide to business and investment plans for private and corporate investors.

SOURCE OF DATA:

The data used in this work were obtained from seven Nigeria different companies financial statement, which can also be verified with Nigeria Stock Exchange (NSE).

The companies are Nigeria Breweries, Total Nigeria PLC, Nestle Nigeria PLC, Dangote Cement group of company, Zenith Bank PLC, Guinness Nigeria and united Bank of Africa PLC

(UBA). The financial statement of these companies was assessed for five years, from 2012-2016, to know their performances before one could invest or continue to invest in them or to diversify.

Information from 2016 different companies Financial Statements

Table 4.1

Companies	Nigerian Breweries (XA)	Guinness (XB)	Nestle (XC)	Total PLC (XD)	Dangote Cement (XE)	Zenith Bank (XF)	U.B.A (XG)
Dividend/Return (₦)	4.60	3.20	38.96	17.00	8.50	2.02	0.75
Risk (%)	0.55	0.70	0.82	0.83	0.48	0.85	0.87

In this problem, objective function is in terms of separate companies while the constraint is in form of risks and dividends associated with the companies.

The dividend is summed up to get the total return from the investment. The ratio of each stock to the total return is also calculated which gives us the contribution of each stock to the portfolio following the Markowitz theory on a modern portfolio as stated in section 3.1. We also took the sum of the risk to know the actual or total risk that is being faced by the portfolio. Later, the ratio of each risk to total risk is taken to know how much risk each security is contributing to the portfolio.

Total Return= 4.60 + 3.20 + 38.96 + 17 + 8.50 + 2.02 + 0.75

= 75.03.

Using the total return as computed above and the dividend from table 4.1, we obtain our ratio as follows:

$$a = \frac{4.60}{75.03} = 0.06 \qquad b = \frac{3.20}{75.03} = 0.04$$

$$c = \frac{38.96}{75.03} = 0.52 \qquad d = \frac{17.00}{75.03} = 0.23$$

$$e = \frac{8.50}{75.03} = 0.11 \qquad f = \frac{2.02}{75.03} = 0.03 \qquad \text{(i)}$$

$$g = \frac{0.75}{75.03} = 0.01$$

Total Risk Factor= 0.55 + 0.70 + 0.82 + 0.83 + 0.48 + 0.85 + 0.87
= 5.10

Using the total risk as computed above and the risk as given in table 4.1 we obtain the risk ratio as follows:

$$a = \frac{0.55}{5.10} = 0.11 \qquad b = \frac{0.70}{5.10} = 0.14$$

$$c = \frac{0.82}{5.10} = 0.16 \qquad d = \frac{0.83}{5.10} = 0.16$$

$$e = \frac{0.48}{5.10} = 0.09 \qquad f = \frac{0.85}{5.10} = 0.17$$

$$g = \frac{0.87}{5.10} = 0.17 \qquad \text{(ii)}$$

$$C_j = \frac{Return\ in\ i}{Risk\ in\ i}, \text{ where } j = A, B, C, D, E, F and\ G.$$

and

$$i = a, b, c, d, e, f and\ g.$$

$$C_A = \frac{return\ in\ a}{risk\ in\ a} \rightarrow \frac{0.06}{0.11} = 0.55$$

$$C_B = \frac{return\ in\ b}{risk\ in\ b} \rightarrow \frac{0.04}{0.14} = 0.29$$

$$C_C = \frac{return\ in\ c}{risk\ in\ c} \rightarrow \frac{0.52}{0.16} = 3.25$$

$$C_D = \frac{return\ in\ d}{risk\ in\ d} \rightarrow \frac{0.23}{0.16} = 1.44$$

$$C_E = \frac{return\ in\ e}{risk\ in\ e} \rightarrow \frac{0.11}{0.09} = 1.22 \qquad \text{(iii)}$$

$$C_F = \frac{return\ in\ f}{risk\ in\ f} \rightarrow \frac{0.03}{0.17} = 0.18$$

$$C_G = \frac{return\ in\ g}{risk\ in\ g} \rightarrow \frac{0.01}{0.17} = 0.06$$

Following from the above calculations of risk and return involved in the seven companies used in this study, we therefore adopted the ratios as yardstick for the computation of simplex method model. The model is illustrated below as:

$$\text{Max } Z = 0.55X_A + 0.29X_B + 3.25X_C + 1.44X_D + 1.22X_E + 0.18X_F + 0.06X_G$$

Subject to:

$$0.06x_A + 0.04x_B + 0.52x_c + 0.23x_D + 0.11x_E + 0.03x_F + 0.01x_G \leq 75.03$$

$$0.11x_A + 0.14x_B + 0.16x_C + 0.16x_D + 0.09x_E + 0.17x_F + 0.17x_G \leq 5.10$$

$$x_A, x_B, x_C, x_D, x_E, x_F, x_G \geq 0$$

Solving the Primal,

Table 4.2

CB_i	c_j	0.55	0.29	3.25	1.44	1.22	0.18	0.06	0	0		
	Basic Variable	x_A	x_B	x_c	x_D	x_E	x_F	x_G	s_1	s_2	Solution variable	Ratio
0	s_1	0.06	0.04	0.52	0.23	0.11	0.03	0.01	1	0	75.03	144.29
0	s_2	0.11	0.12	0.16	0.16	0.09	0.17	0.17	0	1	5.10	31.88←
	z_j	0	0	0	0	0	0	0	0	0	0	
	$c_j - z_j$	0.55	0.29	3.25	1.44	1.22	0.18	0.06		0		
				↑								

From table 4.2 it can be observed that S_2 is the leaving variable and X_C is the entering variable

Solving table 4.2 gives rise table 4.3,

Table 4.3

CB_i	c_j	0.55	0.29	3.25	1.44	1.22	0.18	0.06	0	0	
	$B.V$	x_A	x_B	x_c	x_D	x_E	x_F	x_G	s_1	s_2	s_v
0	s_1	-0.3	-0.42	0	-0.29	-0.18	-0.52	0.54	1	3.25	58.45
3.25	x_c	0.69	0.88	1	1	0.56	1.06	1.06	0	6.25	31.88
	z_j	2.24	2.86	3.25	3.25	1.82	3.45	3.45	0	20.31	103.61
	$c_j - z_j$	-1.69	-2.57	0	-1.81	-0.6	-3.27	-3.39	0	-20.31	

From table 4.3, the solution is optimal and feasible, which satisfied the optima and constraint condition.

$z = ₦103.61,$

$x_c = 31.88, x_A = 0, x_B = 0, x_D = 0, x_E = 0, x_F = 0, x_G = 0.$

To verify,

Max

$$z = 0.55(0) + 0.29(0) + 3.25(31.88) + 1.44(0) + 1.22(0) + 0.18(0) + 0.06(0) = 103.6$$

From the result we see that if we invest 31.88 that we are going to get 103.61

For selection of second best

Table 4.4

C_{BJ}	C_J	1	0.56	2.47	2.18	3	0.1	0	0	
	B_V	X_A	X_B	X_D	X_E	X_F	X_G	S_1	S_2	S_V
0	S_1	0.13	0.09	0.47	0.24	0.06	0.02	1	0	36.07
0	S_2	0.13	0.16	0.19	0.11	0.20	0.20	0	1	4.28
	Z_J	0	0	0	0	0	0	0	0	
	$C_J - Z_J$	1	0.56	2.47	2.18	3	0.1	0	0	

We did not get optimal solution here because $C_J - Z_J \geq 0$, but for it to be optimal solution $c_j - z_j \leq 0$

We continue until we get a solution that is optimal

Table 4.5

C_{BJ}	C_J	1	0.56	2.47	2.18	3	0.1	0	0	
	B_V	X_A	X_B	X_D	X_E	X_F	X_G	S_1	S_2	S_V
0	S_1	0.24	0.05	0.41	0.21	0	-0.04	1	0.3	34.81
3	X_F	0.65	0.8	0.95	0.55	1	1	0	5	21.4
	Z_J	1.95	2.4	2.85	1.65	3	3	0	15	
	$C_J - Z_J$	-0.95	-1.84	-0.38	0.53	0	-2.9	0	-15	

We have not gotten our optimal solution because in $C_J - Z_J$ row, we still have numbers greater than zero.

Table 4.6

C_{BJ}	C_J	1	0.56	2.47	2.18	3	0.1	0	0	
	B_V	X_A	X_B	X_D	X_E	X_F	X_G	S_1	S_2	S_V
0	S_1	-0.21	-0.26	0.05	0	-0.38	-0.42	1	-1.61	26.64
2.18	X_E	1.18	1.45	1.72	1	1.82	1.82	0	9	38.91
	Z_J	2.57	3.16	3.75	2.18	3.97	3.97	0	19.62	84.82
	$C_J - Z_J$	-1.57	-2.6	-1.28	0	-0.97	-3.87	0	-19.62	

From table 4.6, the solution is optimal and feasible, which satisfied the optimal and constraint condition.

$\therefore z = ₦84.82$ and $X_E = ₦38.91$

Selection of Third best

Table 4.7

C_{BJ}	C_J	1.21	0.22	2.82	0.32	0.13	0	0	
	B_V	X_A	X_B	X_D	X_F	X_G	S_1	S_2	S_V
0	S_1	0.17	0.04	0.62	0.07	0.03	1	0	27.57
0	s_2	0.14	0.18	0.22	0.22	0.23	0	1	3.8
	Z_J	0	0	0	0	0	0	0	
	$C_J - Z_J$	1.21	0.22	2.82	0.32	0.13	0	0	

The solution is not optimal; we then apply simplex method in solving it in other to get optimal solution.

Table 4.8

C_{BJ}	C_J	1.21	0.22	2.82	0.32	0.13	0	0	
	B_V	X_A	X_B	X_D	X_F	X_G	S_1	S_2	S_V
0	S_1	-0.23	-0.47	0	-0.55	-1.62	1	-2.82	16.86
2.8	x_D	0.64	0.82	1	1	1.05	0	4.55	17.27
	Z_J	1.80	2.31	2.82	2.82	2.96	0	13.10	48.70
	$C_J - Z_J$	-0.59	-2.09	0	-2.51	-2.83	0	-13.10	

Looking at the table 4.8 we see that the problem has gotten it optimal solution because $C_J - Z_J \leq 0$

Then, $z = ₦48.70$ and $X_D =$ ₦ 17.27

Conclusion

We have reiterated that this paper has interrogated portfolio selection and optimal investment in Nigeria stock market. This work took the ratios of variables which is risk and return got from seven companies floating in Nigeria to form the model equation which we used in calculation of simplex method. This simplex method is used in determining the optimal solution, which will enable investors to choose the best three companies to invest in, and how much to invest in order to minimize the risk and maximizing their return while investing.

From our results, we see that the first place an investor should invest in among seven companies selected is x_C which is **Nestle. And** if the investor should invest ₦31.88 in the product that he will be getting the return of ₦ 103.61

Again, the second place to invest in should be in x_E which is Dangote Cement, that when an investor invested ₦38.91 will be having a return of ₦84.82.

Then, the third is Total PLC which is x_D, in which an investor can get ₦48.70 in investing

₦ 17.27.

The reason for chosen more than one product, is to help the investors to diversified, in other to minimize the risk while maximizing their returns.

This paper recommended the need for investors to resort to optimal calculations before choosing likely stocks to invest in. This will enhance profit maximization and while reducing the risk associated with stock investment.

References

Antoine, & Bertille. (2012). Portfolio selection with estimation risk: A test-based approach. *Journal of Financial Econometrics*, 10(1): 164-197.

Bai, Z., Huixia, L., & Wing-Keung, W. (2009). Enhancement of the applicability of Markowitz's portfolio optimization by utilizing random matrix theory. *mathematical financial studies*, pp. 19,639-667.

Chandra, S., Dharmaraja, S., Aparna, M., & Khemchandani, R. (2013). *Financial Mathematics An Introduction*. New Delhi: Narosa Publishing House.

Dhaene, J., Vanduffel, S., Goovaerts, M., Kaas, S., & Vyncke, D. (2005). Comonotonic Approximation for Optimal Portfolio Selection Problem. *Journal of Risk and Insurance/ vol 72*, 1-45.

Harry, M. (1952). Portfolio Selection. *the Journal of Finance*.

Lee, C. (1977). Functional form, skewness effect and the risk return relationship. *Journal of Financial and Quantitative Analysis*, 12:55.

Lichtenberger, R., & Kraus, A. (1976). skewness preference and the diversification. *Journal Finance*, 31: 1085-1099.

Mark, A.L. (2002). Measures of Risk- Variance and Standard Deviation. Business Finance Online, 1-2.

Mbah, G.C.E, & Onwukwe, L. (2016). Linear Programming method for portfolio selection and optimal financial investment in a developing economy. *International Journal of Applied Research*, 2(12): 338-342.

Megginson, W. (1996). A historical overview of research in finance. *Journal of Finance*, 39(2): 323-346.

Nwidobie,M. B. (2013). Mean Variance Analysis and efficient portfolio selection in Nigeria Capital market. *covenant Journal of Business and Social Sciences*, 1-12.

Ogryczak, W. (2000). Multiple criteria linear programming model for portfolio selection. *Annals of operation Research*, 143-162.

Onwukwe, L. (2016). Portfolio selection and Optimal investment. Ph.D Project, pp.10-30,

Sharma, J. K. (2016). *Operations Research Theory and Application*. India: Trinity.

Von Neumann, J., & Morgenstern, O. (1947). *Theory of games and Economic Behavior 2nd ed.* Princeton, NJ: Princeton university press.

Zkjadoon. (2016, April 21). *Business study notes*. Retrieved from

http://www.businessstudynotes.com/others/intoduction-to-business/define-shares-explain-the-types-of-shares-in-detail/ 60